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Adaptive Telemetry Systems

Status Report

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INTRODUCTION

The research effort covered by this report has been directed towards providing a more thorough understanding of the possibilities and limitations of utilizing feedback telemetry systems to minimize the required spacecraft power supply energy.

A survey of the pertinent literature concerning feedback communication systems is given. None of these papers consider the feasibility of a tradeoff between spacecraft energy and error probability. The assignment of loss (or cost) functions to both the expended energy and the probability of error permits system optimization to be reduced to a minimization of the expected value of the overall loss function. A brief description of potential loss functions is included.

The error probability for a phase coherent sequential detection feedback telemetry system is calculated and the optimum word length for a memoryless feedback system is also presented. Also, the performance of a truncated (finite maximum number of retransmissions) memoryless feedback system is determined and the reduction in the required energy is calculated for a typical system.

BACKGROUND

Schwartz, Harris, Metzner, Morgan, Hauptschein, and Chang have written a series of interrelated papers investigating the use of feedback in communication systems. (Much of the content of these papers is summarized in Chapter 10 of L.S. Schwartz's book[12].) They classify feedback communication systems into two broad areas as follows:

1) Decision feedback: Either a null-zone decision system or an error detecting code is used with the result that the receiver decides whether or not the message should be retransmitted.

2) Information feedback: The receiver decodes the message and sends the decoded message back to the transmitter via a feedback path. The transmitter then decides whether or not the message was received correctly and proceeds accordingly.

These authors treat binary transmission using null-zone reception techniques. The null-zone receiver uses two thresholds and demands that a message be repeated only if the received message falls in the null-zone. In this type of feedback system the probability of making an error in reception is made smaller by increasing the null-zone thresholds, but this is done at the expense of decreased information rate. For example, according to Schwartz[12], with a SNR of 6 db a non-feedback system will have an information rate of 0.84 and an error probability of 0.06, while a null-zone feedback system adjusted for optimum rate has an information rate of 4 percent higher and an error probability of 0.01.

This group of authors also considered the problem of minimizing the average transmission time (for specified error probability) by adjusting the null-zone thresholds on subsequent transmissions of a repeated message, obtaining the result that the uniform-null, non-truncated system is optimum. They also show that for truncated discarding decision-feedback systems an optimum procedure exists for adjusting the null-zone levels on subsequent transmissions; however, the improvement in the error probability is small and, therefore, nonuniform adjustment of the thresholds in truncated systems is not recommended.

The effects of cumulative decision feedback (sequential detection - the transmitter keeps repeating the transmitted message until the receiver indicates the message is no longer ambiguous) to combat "fast-fading" in smaller-multipath reception was investigated by Chang et al[1], assuming the SNR was Raleigh distributed and the noise additive Gaussian. They show, for instance, that for SNR = 0 db and $P_e = 10^{-6}$, the cumulative decision feedback system is better than the discarding decision feedback system by a factor of about 18 with respect to normalized transmission time.

Metzner and Morgan[8] consider using feedback together with long code groups. By emphasizing error detection rather than concentrating exclusively on error correction a reduction in error probability of several orders of magnitude is achieved while only slightly increasing the transmission time. The codes used are Hamming codes.

Information feedback systems allow incorrectly decoded messages to be subsequently corrected since the transmitter knows when errors are made. Chang[2] describes a single error correcting procedure (iterative discarding) based on the ability to transmit an erasure symbol. Schwartz claims that information feedback is better than decision feedback provided the SNR is about 3 db or greater, while decision feedback is better for small signal-to-noise ratios, there being a sharp threshold effect. A better information rate than that obtainable with iterative discarding (errors are immediately corrected) is obtainable by having the transmitter transmit synchronous error messages consisting of coded error positions and corrections. Also, truncation problems may arise in finite memory information feedback systems due to the possibility of long delays before a message is correctly received. Harris, Morgan, Hauptschein, and Schwartz[4] compare information and decision feedback systems (two bits per word) for SNR's of 0 and 6 db and for different truncation lengths. The data is exhibited by plotting error probabilities versus average normalized transmission times.

Turin, in two papers[14,15], obtained a partial solution to the problem of signal design for binary signals corrupted by additive white Gaussian noise in the forward channel by using information feedback. The feedback quantity (using a noiseless delayless channel) was the a posteriori uncertainty function. Sequential and non-sequential (fixed transmission time) detection were considered, and for a fixed error probability the transmission time was minimized. The class of binary signals considered was of the form $S_{\pm}(t) = \pm U_{\pm}(y)\sigma(t)$, where y is the uncertainty. Exact solutions were obtained for the two cases where the ratio of peak-to-average power was zero and unity.

Horstein[7] extends Turin's analysis by considering arbitrary peak-to-average power ratios.

Viterbi[16] has investigated binary and M-ary continuous sequential decision feedback systems over the Gaussian channel. For the binary case the transmitter uses PSK with sinusoidal waveforms, and for the M-ary case FSK sinusoidal signals are used. For the binary case the SNR may be reduced by a factor of four at low SNR's compared to that of a non-feedback channel operating at the same error probability and rate.

In the M-ary case the improved performance exhibited by the sequential decision system compared to the non-sequential system is expressed in the exponent of the error bound.

$$P_e = KM^{-(C/\overline{R})\alpha} \text{ with feedback and}$$

$$P_e = K'M^{-(C/\overline{R})\alpha'} \text{ without feedback}$$

where $\overline{R}$ is average rate of transmission.(In sequential detection the time to transmit each signal is a random variable). Viterbi shows that $\frac{\alpha}{\alpha'}$ ranges between two and four while $\overline{R}/C$ varies between one-fourth and one.

LOSS FUNCTIONS

In analyzing feedback communication systems the primary interest of most investigators has been in minimizing the probability of error. In more realistic situations one is also interested in minimizing the energy used in transmission by the satellite and in maximizing the information rate. These are all conflicting requirements so that in actual systems a compromise has to be made according to the designer's a priori knowledge of the cost or loss due to an error, the energy used, or the information rate. To formulate the problem mathematically, a loss function conforming to this a priori knowledge will be defined and designated as $L(p_e, E, r)$ where p_e is the probability of error, E is the energy used and r is the information rate. Having chosen a loss function it will then be possible to determine the parameters of any given system to minimize the expected loss. In addition, different systems can be compared in terms of their expected loss.

Basically there are two general types of feedback communication systems: decision feedback systems and information feedback systems. In decision feedback schemes the ground receiver examines the received signal and decides whether to accept it as correct or reject it and ask the satellite to retransmit. This decision can be based, for example, on the use of an error detecting code, knowledge of the signal-to-noise ratio at the receiver, or by comparison of the received signal with a predicted signal. In information feedback schemes the received signal or some characterization of the received signal is retransmitted to the satellite. The satellite then decides what course of action to take. Decision feedback has the advantage that the receiver specifies the transmission format. In information feedback the receiver must estimate the format from the received signal, thus introducing additional probability of error.

An investigation is presently being conducted to determine an optimum class of loss functions. The following decision feedback scheme is given as an example.

The ground receiver estimates the signal-to-noise ratio and on this basis requests that the satellite retransmit the message. Assume that the message is transmitted k times. It will also be assumed that the energy for transmission of a one is the same as for a zero. Therefore, the energy used in transmission would be STk where S is the average signal power and T is the message length. The receiver will be assumed to be a maximum likelihood receiver, and the cases of both sequential and nonsequential detection will be investigated. As a first step the loss function could be chosen as $L(p_e, E) = A\ p_e + STk$ where A is an arbitrary constant.

A more sophisticated loss function is given by: $L = \frac{\bar{k}\beta}{(1-p_e{}^{k_m})^{1/n}}$ where $\bar{k}$ is the expected number of transmissions, k_m is the maximum allowable number of transmissions, n is the number of bits per word, p_e is the word error probability, and β is the ratio of energy per bit to noise power spectral density. An even better loss function would depend on the energy remaining in the satellite's battery rather than the cumulative energy used. For example, if E_o is the initial battery energy, p(t) is the power supplied to the battery from the solar cells, and $s(t)$ is the signal power, then the remaining energy E_r is

$$E_r = E_o + \int_{-\infty}^{t} p(t)dt - \int_{-\infty}^{t} s(t)dt.$$

A formulation of this type would allow one to choose $s(t)$ to optimize the performance when the satellite is encountering periods of both light and darkness.

ERROR PROBABILITY FOR FEEDBACK SYSTEM EMPLOYING SEQUENTIAL DETECTION

Consider the calculation of the error probability for a sequential detection phase coherent system employing feedback in which n bits of information are sent over a Gaussian channel using 2^n orthogonal signals of duration Δ. At the end of Δ seconds the receiver decides which message was sent and relays the decision back to the transmitter using a noiseless (i.e., error free) feedback channel. The transmitter then sends a confirm signal if the correct decision was made and otherwise sends a reject signal followed by a retransmission of the entire message, again taking Δ seconds to do so. The probability of an error after the first transmission is

$$P_{e_1} = 1 - \int_{-\infty}^{+\infty} f\,(x - \sqrt{E_s})\ \left[\int_{-\infty}^{x/\sqrt{N_o}} f(y)\ dy\right]^{2^n-1} dx$$

where E_s = energy in each of the orthogonal signals and

$$f(\alpha) = \frac{1}{\sqrt{\pi N_o}}\ e^{-\alpha^2/N_o}$$

It is customary to use β as the independent variable.

$$\beta = \frac{1}{n}\,\frac{E_s}{N_o} = \frac{S}{N_o}\left(\frac{\Delta}{n}\right) = \frac{ST}{N_o}$$

$$S = \text{average power} = \frac{E_s}{\Delta}$$

$$T = \text{time per bit} = \frac{1}{n}\Delta.$$

This is done to provide a means of comparison between orthogonal systems with different values of n.

Suppose after the first transmission an error is made and the transmitter retransmits the signal after verification of the error. This eliminates the possibility of transmission of the signal previously decoded and does not in any way effect the relative (to each other) probabilities of the remaining 2^n-1 choices. Therefore, the error probability is now:

$$P_{e_2} = 1 - \int_{-\infty}^{+\infty} f(x-\sqrt{2E_s}) \left[\int_{-\infty}^{x/\sqrt{N_o}} f(y)\, dy\right]^{2^n-2} dx$$

The overall word error probability is now equal to the product $P_{e_1}P_{e_2}$, and the expected value of the required transmission time is:

$$\Delta(1 + P_{e_1})$$

Also, the transmitted energy is proportional to: $(1 + P_{e_1})$.

The word error probabilities P_{e_1} and P_{e_2} can be considered to be functions of n and β. For large values of n, it follows that

$$P_{e_1}(n,\beta) \approx P_{e_2}(n,\beta) = P_e(\beta)$$

Therefore, the overall error probability is approximately equal to:

$$P_e(\beta)\ P_e(2\beta)$$

For smaller values of n, the above represents an upper bound for the overall error probability.

OPTIMUM WORD LENGTH FOR MEMORYLESS FEEDBACK SYSTEM

Consider the transmission of a total of θ bits of information via a Gaussian channel using n-bit orthogonal codes, and assume the existence of a noiseless feedback channel which allows the transmitter to know what decision the receiver has made about the transmitted word. It is assumed that the transmitter repeats transmission of words that are received in error until they are correctly received and also that the receiver examines only each new signal of length n bits in the decision process, and discards the previously transmitted signal completely. (The result is easily modified if sequential detection is employed. See previous section.) It is also assumed that some method exists to allow the transmitter to inform the receiver to accept the correctly decoded message or reject the incorrectly decoded message. This may be done by using a separate channel, or by data format, or by interleaving accept or reject signals between the data words, provided these accept or reject signals are essentially errorfree.

It is of interest to know how the length of the code words affects the performance of such a system. Assume a total of θ bits of data that must be transmitted with no errors. Let the longest code word to be considered have length N_{max}, and for ease in comparing results assume $\theta = (N_{max})!$ so that θ is divisible by every integer j, $1 \leq j \leq N_{max}$. Let $\beta = ST/N_o$, the ratio of energy per bit to noise power spectral density, be fixed. That is, the transmitted energy per bit (whether accepted or rejected) and the channel noise variance are fixed. The total energy required to transmit without error the θ bits is

$$E_{total} = \begin{bmatrix}\text{number of}\\ \text{words}\end{bmatrix} \begin{bmatrix}\text{word length}\\ \text{in bits}\end{bmatrix} \begin{bmatrix}\text{expected no. of trans.}\\ \text{to get a word correct}\end{bmatrix} \begin{bmatrix}\text{energy}\\ \text{per bit}\end{bmatrix}$$

$$= [\tfrac{\theta}{n}]\,[n]\,[\bar{k}]\,[ST]$$

$$= \theta ST\,\bar{k}$$

where $\bar{k}$ is the expected value of the number of trials necessary to transmit each word correctly. If the probability of incorrectly decoding a transmitted word of length n is given by $P_e(n)$, then the probability density function of k is given by:

$$f(k) = [P_e(n)]^{k-1}[1-P_e(n)]\,,\quad k = 1, 2, \ldots, \infty$$

Therefore:

$$\bar{k} = \sum_{k=1}^{\infty} k[P_e(n)]^{k-1}[1-P_e(n)] = \frac{1}{1-P_e(n)}$$

or if $P_e(n) \ll 1$ then

$$\bar{k} \approx [1 + P_e(n)].$$

Upon examining a plot of $P_e(n)$ versus β it is seen that, if $\beta > 1$, then $P_e(n)$ is a monotone decreasing function of n for fixed β, and E_{total} will also decrease an n increases. On the other hand if $\beta < 1$ then $P_e(n)$ and E_{total} will increase with n. Hence, for $\beta > 1$ we should use the longest codes available, and for $\beta < 1$ we should use the shortest codes available.

PERFORMANCE OF TRUNCATED MEMORYLESS FEEDBACK SYSTEM

Consider the performance of a memoryless system employing an errorfree feedback channel when the number of retransmissions is arbitrarily truncated. Assume that the system is transmitting a 10-bit orthogonal code. If β remains fixed each time a word is transmitted, so that the probability of an error on each reception is fixed, and if the maximum number of trials is k_m, then the probability that a word is not received correctly after the k_m trials is:

$$[P_e(10)]^{k_m}$$

The total energy required, on the average is

$$\sum_{j=0}^{k_m-1} ST\ [P_e(10)]^j$$

and truncated systems of this type may be compared with others by specifying a fixed word error probability. For example, if a 10-bit code word is to be received with an error probability of 10^{-5}, then a sequential memoryless system with a truncation length of three trials must have a word error probability for each trial of $P_e(10) = (10^{-5})^{1/3} \approx 0.022$ which yields $\beta = ST/N_o \approx 1.6$. The average energy required is

$$1.6(N_o)\ (1 + .022 + (.022)^2 + \cdots) \approx 1.63(N_o).$$

A non-sequential system with the same error probability will have $\beta = 4$ so that the sequential system with truncation length of 3 and error probability of 10^{-5} is 60 percent more efficient from an energy standpoint.

Other sequential decision techniques may be used. For example, consider a memoryless truncated scheme in which β

is increased on subsequent transmissions. Let n = 10 bits and let the truncation length be three trials, but now assume the time per word is doubled in each subsequent transmission so that the times per word are T_o, $2T_o$, and $4T_o$. If $ST_o/N_o = 1.8$ initially, then the probabilities of incorrectly receiving the three orthogonally coded words are approximately 10^{-2}, 4×10^{-5}, and 3×10^{-8}. The truncated system will have an error probability of: $10^{-2} \times 4 \times 10^{-5} \times 3 \times 10^{-8} = 12 \times 10^{-15}$ and will use an average energy of

$$(1.8 + 3.6 \times 10^{-2} + 7.2 \times 4 \times 10^{-5})\ (N_o) \approx 1.8(N_o)$$

A single-shot transmitter-receiver opreating in the same channel sending 10-bit orthogonal code words at the same error probability will need about 15 times the energy per code word. Thus, it is clearly evident that sequential systems employing errorfree feedback can provide very low error-probabilities while at the same time minimizing spacecraft energy consumption.

Substantial energy reduction is possible even if one uses a one-bit word for the accept or reject signals which are interleaved with the n-bit code words. For example, consider a memoryless system sending 10-bit orthogonal code words and using a one-bit accept or reject code. Let $P = P_e(10)$ be the probability of incorrectly decoding a 10-bit word on each trial, and let $Q = P_e(1)$ be the probability of making an error on the one-bit signal. With a truncation length of two (so that the one-bit message is sent only once) the overall probability of error is

$$P(\text{error}) = (1-P)\ QP + P^2\ (1-Q) + PQ.$$

Let T_1 be the time per bit allocated to the code word and let T_2 be the time allocated to the accept or reject bit, and assume the average power is equal to S throughout. The average energy required to transmit the 10-bit code word will be:

$$[ST_1 \ (1 + P + Q - 2PQ) + ST_2] \approx S(T_1 + T_2) \quad \text{if } P,Q << 1.$$

For example, if $P = 10^{-2} = P_e(10)$ and $Q = 10^{-2} = P_e(1)$, then $\beta_1 = ST_1/N_o \approx 1.8$ and $\beta_2 = ST_2/N_o \approx 5.2$ and we see $P(\text{error}) \approx 3 \times 10^{-4}$ while the expected value of the energy is 23 (N_o). A one-shot 10-bit orthogonal code system with the same error probability would have an average energy of 30 (N_o) so that the truncated system in this example is 23 percent more efficient in its use of energy. For systems like this the energy allocated to the accept or reject signal must be kept small. If not, the non-sequential system will be better. For example, with $n = 10$, $P = 10^{-2}$, $Q = 10^{-4}$, we see that the expected value of energy is 33 (N_o) and $P(\text{error}) \approx 10^{-4}$, whereas a non-sequential system with $P_e(10) = 10^{-4}$ would require only $ST = 30$ (N_o).

SELECTED BIBLIOGRAPHY

1. S.S.L. Chang, B. Harris, and K.L. Morgan, "Cumulative Binary Decision-Feedback Systems," Proc. NEC, vol. 14, pp. 1044-1057, 1958.

2. S.S.L. Chang, "Theory of Information-Feedback Systems," IRE Transactions on Information Theory, vol. IT-2, pp. 29-40, September 1956.

3. P.E. Green, Jr., "Feedback Communication Systems," in E.J. Baghdady, Lectures on Communication System Theory, McGraw-Hill Book Co., Inc., New York, 1961.

4. B. Harris, A. Hauptschein, K.C. Morgan and L.S. Schwartz, "Binary Communication Feedback Systems," Transactions AIEE, vol. 77, pp. 960-969, January 1959.

5. B. Harris, A. Hauptschein, and L.S. Schwartz, "Optimum Decision-Feedback Systems," 1957 IRE National Convention Record, Part 2, pp. 3-10, 1957.

6. B. Harris and K.C. Morgan, "Binary Symmetric Decision Feedback Systems," Transactions AIEE, vol. 77, pp. 436-443, September 1958.

7. M. Horstein, "On True Design of Signals for Sequential and Nonsequential Detection Systems with Feedback," IEEE Transactions on Information Theory, vol. IT-12, pp. 448-455, October 1966.

8. J.J. Metzner and K.C. Morgan, "Coded Binary Decision-Feedback Communication Systems," IRE Transactions on Communications Systems, vol. CS-8, pp. 101-112, June 1960.

9. R.W. Rochelle, "Pulse Frequency Modulation Telemetry," NASA Technical Report R-189, January 1964.

10. L.S. Schwartz, "Feedback for Error Control and Two-Way Communication," IEEE Transactions on Communications Systems, vol. CS-11, pp. 49-56, March 1963.

11. L.S. Schwartz, "Some Recent Developments in Digital Feedback Communication Systems," IRE Transactions on Communications Systems, vol CS-9, pp. 51-57, March 1961.

12. L.S. Schwartz, Principles of Coding, Filtering, and Information Theory, Spartan Books, Inc., Baltimore, 1963.

13. J.P. Strong, III, T. V. Saliga, "Comparison of Phase-Coherent and Non-Phase-Coherent Coded Communications," NASA Technical Note D-3079.

14. G.L. Turin, "Comparison of Sequential and Nonsequential Detection Systems with Uncertainty Feedback," IEEE Transactions on Information Theory, vol. IT-12, pp. 5-8.

15. G.L. Turin, "Signal Design for Sequential Detection Systems with Feedback," IEEE Transactions on Information Theory, vol. IT-11, pp. 401-408, July 1965.

16. A.J. Viterbi, "The Effect of Sequential Decision-Feedback on Communication over the Gaussian Channel," Information and Control, vol. 8, pp. 80-92, February 1965.

17. A.J. Viterbi, "Improvement of Coherent Communication over the Gaussian Channel by Error-Free Decision Feedback," JPL Space Programs Summary No. 37-23, vol. IV, pp. 179-180, October 1963.